Tearing growth rate of a viscoresistive Harris sheet subject to flow

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Abstract. The MHD spectroscopic *Legolas* code is applied to a viscoresistive Harris current sheet for varying values of resistivity and viscosity, with and without the presence of shear flow. It is shown that whether the background flow has a stabilising or destabilising effect on the tearing instability depends on the specific combination of resistivity and viscosity.

Introduction

Due to its prevalence in a large variety of explosive and impactful phenomena, like coronal mass ejections, the solar wind-magnetopause interaction, and confinement disruption in fusion devices, magnetic reconnection has become crucial to our understanding of plasma behaviour. In this process of breaking and reconfiguring magnetic field lines, the alteration of the magnetic topology results in a conversion of magnetic to thermal and kinetic energy. However, reconnection is regularly observed to occur much faster than the original Sweet-Parker model^(7;8) can account for on its own. Though it is not the only way of triggering reconnection, we here focus on the resistive tearing instability, first described by Furth, Killeen, and Rosenbluth⁽⁴⁾, and analyse how its growth rate depends on the resistivity and viscosity in a current sheet.

Harris current sheet with shear flow

In this study, we adopt the Harris current sheet setup of Li and Ma⁽⁵⁾, which features a reversal of the magnetic field B_0 across the x = 0 plane, without guide field,

$$\boldsymbol{B}_0(x) = B_0 \tanh\left(\frac{x}{a_B}\right) \, \boldsymbol{\hat{e}}_y,\tag{1}$$

in a plasma of uniform density $\rho_0 = 1$. The temperature T_0 is set such that the system is in force balance, i.e. satisfying

$$\frac{\partial}{\partial x} \left(\rho_0 T_0(x) + \frac{1}{2} \boldsymbol{B}_0^2(x) \right) = 0.$$
⁽²⁾

In case a flow profile is included below, this configuration is supplemented with a shear flow profile v_0 of the same form as the magnetic field,

$$\mathbf{v}_0(x) = v_0 \tanh\left(\frac{x}{a_v}\right) \,\hat{\mathbf{e}}_y.$$
 (3)

For all cases considered here, we set $B_0 = 1$, $a_B = 1$, $v_0 = 0.2$, and $a_v = 0.8$.

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Stability analysis with the *Legolas* code

This flow-sheared Harris sheet is then analysed with the spectroscopic Legolas code^(2;3) (v2.0.5, see https://legolas.science), which solves the linearised, compressible MHD equations for all frequencies ω and associated perturbations in density, velocity, temperature, and magnetic field of the form

$$f_1(\mathbf{x},t) = \hat{f}_1(x) \exp[i(k_2 y + k_3 z - \omega t)]$$
(4)

for a specified wave vector $\mathbf{k} = k_2 \, \hat{\mathbf{e}}_y + k_3 \, \hat{\mathbf{e}}_z$. Here, we choose $k_2 = 0.25$ and $k_3 = 0$, unless noted otherwise. In this analysis, we solve the system in the interval $x \in [-15, 15]$ such that the effect of the perfectly conducting boundary conditions is negligible (satisfying $|x_{wall}| \gtrsim 10 a_B)^{(6)}$. The Gaussian spacing function

$$f(x) = p_1 - (p_1 - p_3) \exp\left[-\frac{(x - p_2)^2}{2p_4}\right]$$
(5)

is used to obtain a higher grid point density at the center of the interval, with values $p_1 = 0.75$, $p_2 = 0$, $p_3 = 0.001$, and $p_4 = 2.5$, and we adopt the QR-cholesky solver.

Results and discussion

For this Harris sheet configuration and the aforementioned *Legolas*-specific parameters, we now vary either the resistivity η , the dynamic viscosity μ , or the wave number k_2 , whilst the other two remain fixed. Despite the presence of flow shear, the only unstable mode in the spectrum is the resistive tearing instability since the speed v_0 is significantly sub-Alfvénic everywhere. Additionally, the flow profile is an odd function with respect to the location of the magnetic nullplane (x = 0), such that the tearing instability remains purely imaginary in the case with flow. The tearing growth rates are shown in Fig. 1(a-c) for the flowless case and in Fig. 1(d-f) for the case with flow profile (3). Lastly, Fig. 1(g-i) shows the difference in growth rate between the middle and left columns of Fig. 1.

As expected, panels (a, b) and (d, e) confirm that stronger viscosity results in stronger damping for a certain resistivity. Similarly, a higher resistivity value corresponds to a larger growth rate, except for extremely high resistivities, where the growth rate falls off again in panels (a) and (d), contrary to the literature's analytic scaling laws for incompressible plasmas^(4;1). Finally, panels (c) and (f) show that the wave number of maximal growth varies slightly depending on the resistivity and viscosity, but stays relatively close to the value $k_2 = 0.25$ used in the other panels.

To study the effect of flow in the presence of resistivity and viscosity, we look at the right column. Firstly, flow does not appear to alter the wave number of maximal growth significantly, and



Figure 1: On the left (a-c), the tearing growth rate in the absence of flow. In the middle (d-f), a background flow with $v_0 = 0.2$ and $a_v = 0.8$ is present. On the right (g-i), the difference $\text{Im}(\omega_{\text{flow}}) - \text{Im}(\omega_{\text{no flow}})$ between the middle and left columns. (a, d) Growth rate as a function of η for given values of μ and $\mathbf{k} = 0.25 \,\hat{\mathbf{e}}_y$. (b, e) Growth rate as a function of μ for given values of η and $\mathbf{k} = 0.25 \,\hat{\mathbf{e}}_y$. (b, e) Growth rate as a function of μ for given values of η and $\mathbf{k} = 0.25 \,\hat{\mathbf{e}}_y$. (c, f) Growth rate as a function of $\mathbf{k} = k_2 \,\hat{\mathbf{e}}_y$ for A) $\eta = 10^{-2}$ and $\mu = 10^{-2}$; B) $\eta = 10^{-3}$ and $\mu = 10^{-2}$; C) $\eta = 10^{-2}$ and $\mu = 10^{-3}$; D) $\eta = 2 \times 10^{-3}$ and $\mu = 2 \times 10^{-3}$.

panel (i) reveals that larger growth rates are damped more by the flow shear, though the modification is less than 2%. Secondly, panels (g) and (h) illustrate both the stabilising and destabilising influence of the flow, depending on the physical parameters. Here, panel (h) features a 4-5% growth rate damping for small viscosity values, but a sizeable growth rate increase in the strongly viscous regime. Lastly, panel (g) demonstrates that in strongly viscous media the shear flow will further destabilise the medium whilst it has a mostly stabilising influence in weakly viscous or inviscid media.

Conclusion

In these proceedings we evaluated the interplay between resistivity, dynamic viscosity, and shear flow in a Harris current sheet, and the effect on its tearing instability. In conclusion, the viscosity is found to damp the tearing instability, with stronger viscosity resulting in a stronger suppression. For weakly viscous media, the introduction of shear flow further suppresses the instability. However, in the highly viscous regime the shear flow counteracts the viscosity's effect, and results in a larger growth rate again.

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