

The Legolas code

The *Legolas* code [1,2,3,12] is a magnetohydrodynamic (MHD) spectroscopic code that quantifies all waves and instabilities of a one-dimensionally varying state. It does so by perturbing density, velocity, temperature and magnetic field in the MHD equations and linearizing around an equilibrium of the form

$$\begin{aligned}\rho_0 &= \rho_0(u_1), & \mathbf{v}_0 &= v_{02}(u_1)\hat{\mathbf{e}}_2 + v_{03}(u_1)\hat{\mathbf{e}}_3, \\ T_0 &= T_0(u_1), & \mathbf{B}_0 &= B_{02}(u_1)\hat{\mathbf{e}}_2 + B_{03}(u_1)\hat{\mathbf{e}}_3.\end{aligned}$$

Here, u_1 represents the x-coordinate in Cartesian or the radial coordinate in cylindrical geometry. Then, a Fourier form is assumed for all perturbations f_i in the remaining spatial directions and time,

$$f_i(\mathbf{r}, t) = \hat{f}_i(u_1) \exp[i(k_2 u_2 + k_3 u_3 - \omega t)]$$

where (u_2, u_3) represent (y, z) or (ϑ, z) with corresponding wave numbers (k_2, k_3) and t time with frequency ω . Subsequently, the equations are written in weak Galerkin form. A combination of quadratic and cubic finite elements is used to represent the perturbation amplitudes on the discretized grid. This reduces the system to a generalized eigenvalue problem $\mathbf{A}\mathbf{x} = \omega\mathbf{B}\mathbf{x}$, where $\mathbf{A}$ and $\mathbf{B}$ contain the prescribed equilibrium and wave number information. For N grid points, this problem is solved to yield $16N$ eigenfrequencies in the complex plane with corresponding eigenvectors $\mathbf{x}$ describing the perturbations.

Details:

- The magnetic field perturbation is presented by a vector potential, ensuring that the divergence of the magnetic field perturbation is zero.
- The boundary conditions applied in the u_1 -direction are perfectly conducting walls. In cylindrical geometry, if the radial coordinate goes to zero, an axis regularity condition is applied there instead.
- The code modularly includes background flow, gravity, resistivity, radiative cooling and constant background heating, thermal conduction, viscosity, and Hall physics.
- The code links to multiple eigenproblem solvers. The most commonly used solvers are *QR Cholesky* to obtain the full eigenmode spectrum, *shift-invert* to identify the eigenmodes in a certain region of the spectrum, and *inverse iteration* to find the closest mode to a specified target.

To visualize and interact with *Legolas* data, the *pylbo* Python package is distributed alongside the code. For a Harris current sheet (schematically pictured), the eigenmode spectrum is shown below. The one unstable mode (upper half of the complex plane) is the tearing instability, whose B_x -perturbation is shown on the right. The slow and Alfvén sequences are damped and appear as semi-circles in the lower half of the complex plane. Their ranges are indicated in color on the horizontal axis. On either side of the semi-circular structure, the first fast mode in the forward-backward propagating sequences can be seen.

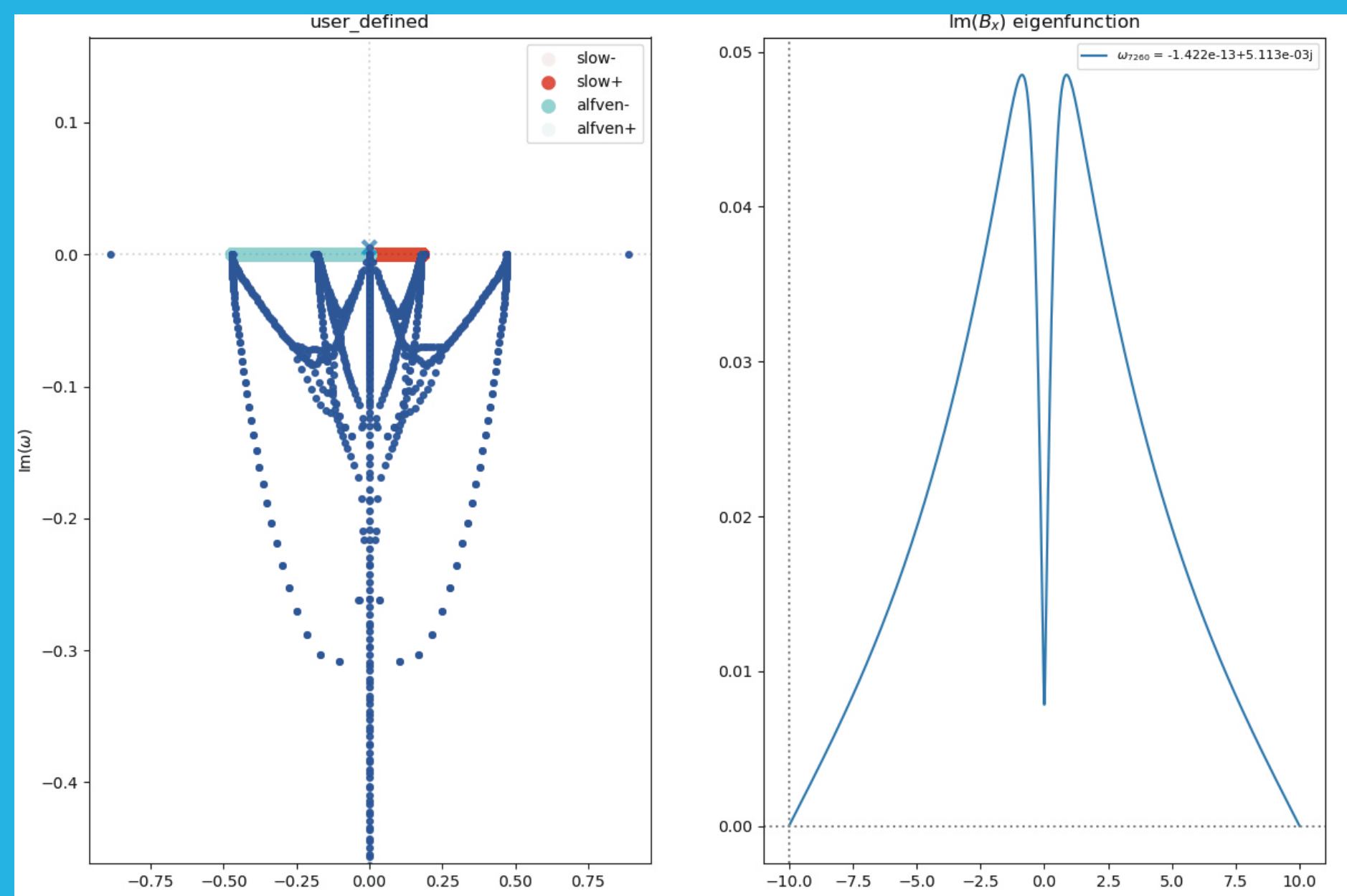
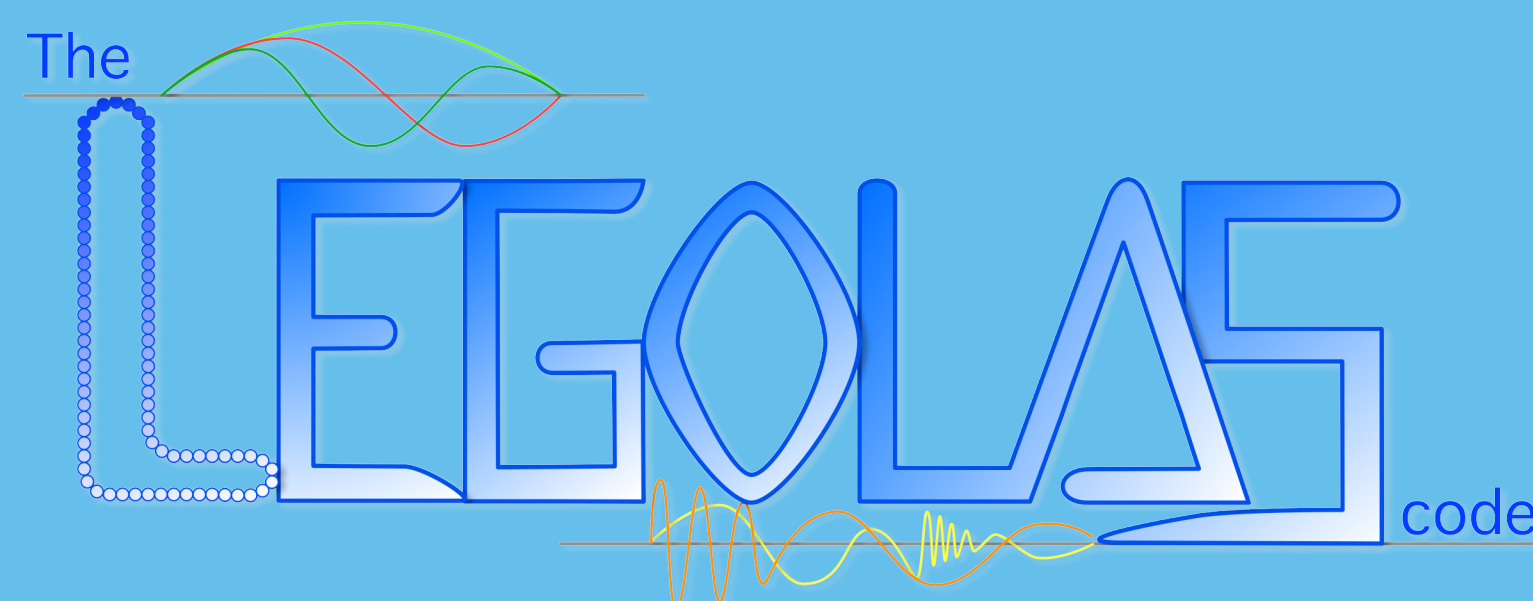
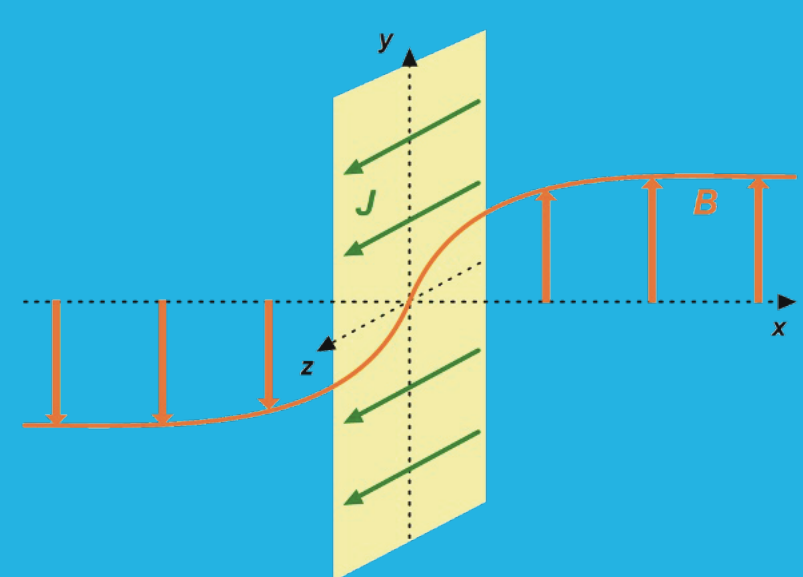


Figure: *Legolas* output for a resistive MHD current sheet. (Left) Eigenmode spectrum. (Right) $\text{Im}(B_x)$ perturbation of the tearing instability.



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Initial Value Problems

Instead of approaching the linearized equations as an eigenvalue problem (see **The Legolas code**), the matrices $\mathbf{A}$ and $\mathbf{B}$ can also be repurposed to solve the initial value problem, i.e. calculate the linear response of a system to a prescribed perturbation. This is implemented in *Legolas-IVP*, a subsystem of the *Legolas* code. In addition to the equilibrium, the user now also specifies an initial perturbation, e.g. a Gaussian pulse, and the code computes the linear response to it.

As an example, consider a coronal loop. Due to optically thin radiative losses, coronal loops can become unstable to thermal instability, which is responsible for the rapid formation of condensations (cool, dense plasma) in the loop. To study this instability, coronal loops are commonly modelled using a one-dimensional hydrodynamic (HD) description, where the curvature of the loop is taken into account only in the gravity term. An example of the model after some time is shown above, with condensations forming at the top of the loop.

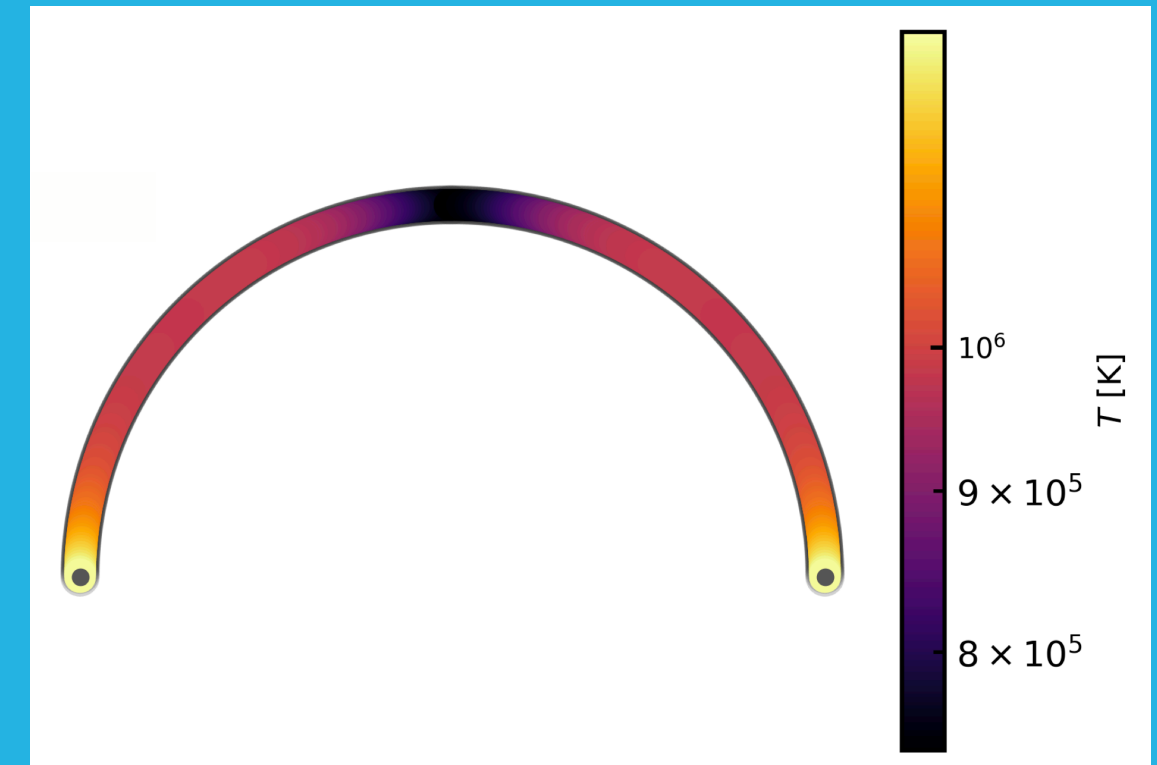


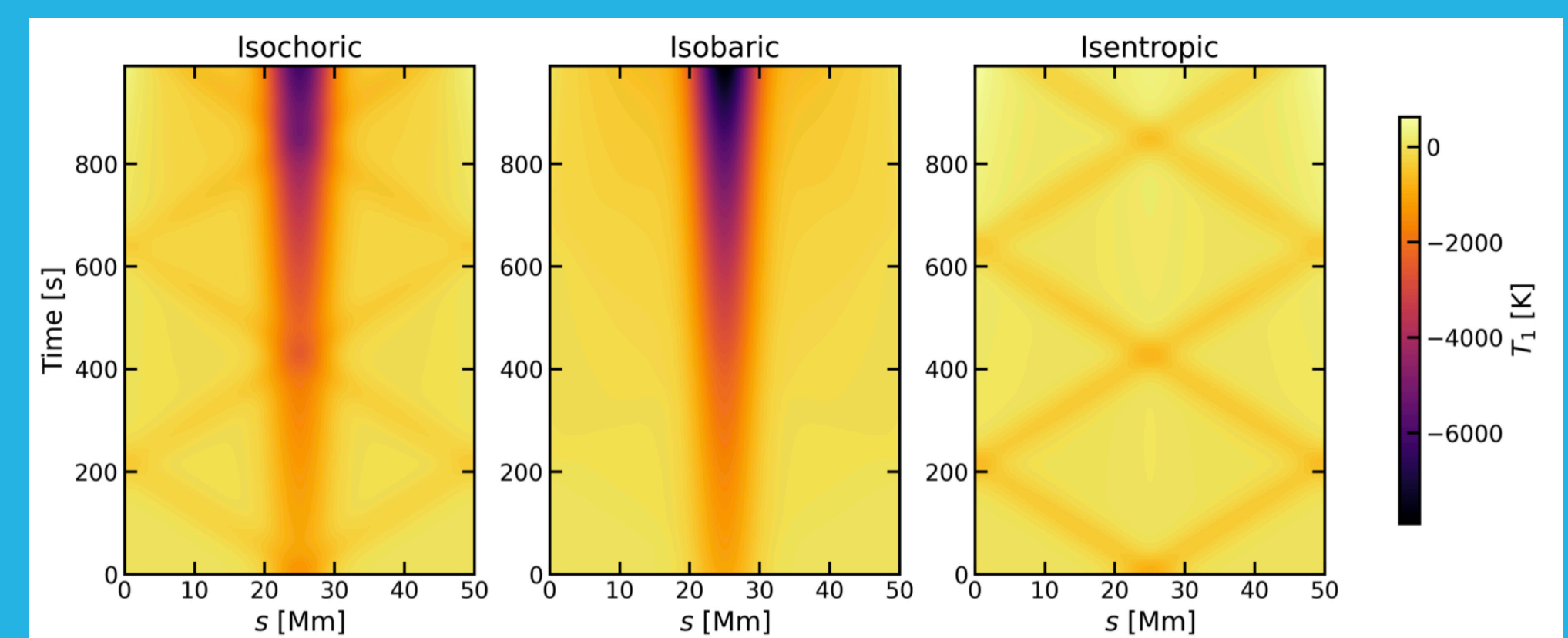
Figure: Condensation formation in a 1D hydrodynamic coronal loop model. [4]

In the spectrum, this instability manifests as a non-oscillatory unstable continuum (the frequency is imaginary and positive). To investigate the excitation of these instabilities, *Legolas-IVP* was initialized with three different Gaussian pulses in [4]: isochoric, isobaric, and isentropic. Their time-distance maps are shown below. Here, it is clear that the isochoric pulse excites both the thermal instabilities, responsible for the looptop cooling, and propagating acoustic waves, which appear as diagonal patterns in the time-distance map and are reflected at the loop footpoints. Contrary to the isochoric pulse, the isobaric pulse only excites the thermal modes and the isentropic pulse only the acoustic waves.

With *Legolas-IVP* it is thus possible to verify which spectral components are excited by specific perturbations to the system. Together with the eigenvalue solver, this offers a more complete picture of system dynamics.

Though the example presented here is solely hydrodynamic, *Legolas-IVP* is available for the full MHD equations from *Legolas* v2.3 onwards.

Figure: A thermally unstable 1D hydrodynamic loop model perturbed with three different initial pulses, as computed with *Legolas-IVP*. [4]



Simulation initial conditions

In many numerical simulations, the initial condition is perturbed with some initial noise, most often in the velocity, to start the development of the system's evolution. In unstable equilibria, this noise excites all eigenmodes, including the instabilities. These instabilities will then grow from the noise until they dominate the system evolution. However, as shown in [5], this start-up phase can be skipped by introducing the linear instability in the initial configuration by perturbing all quantities in a consistent manner. This is shown to the right for a coronal Harris current sheet simulation set up in *MPI-AMRVAC* [11,13].

In the figure below, the energy evolution of five Harris current sheet simulations is tracked. Each simulation used another initial perturbation with an identical total energy perturbation. Case a was perturbed with the most unstable tearing instability, Case b with a pair of forward-backward propagating fast waves, Case c with the superposition of these two, Case d with the magnetic field prescription of [7], and Case e with velocity noise. For this last case, the expected total energy perturbation was used to scale the noise. It is immediately clear that the two simulations that include the tearing instability in their initial perturbation evolve towards current sheet fragmentation the fastest, followed relatively quickly by the simulation initialized with the pair of fast wave eigenmodes. This is followed much later by the simulation using velocity noise, after significant diffusion of the current sheet, and finally by the simulation with analytical prescription for the

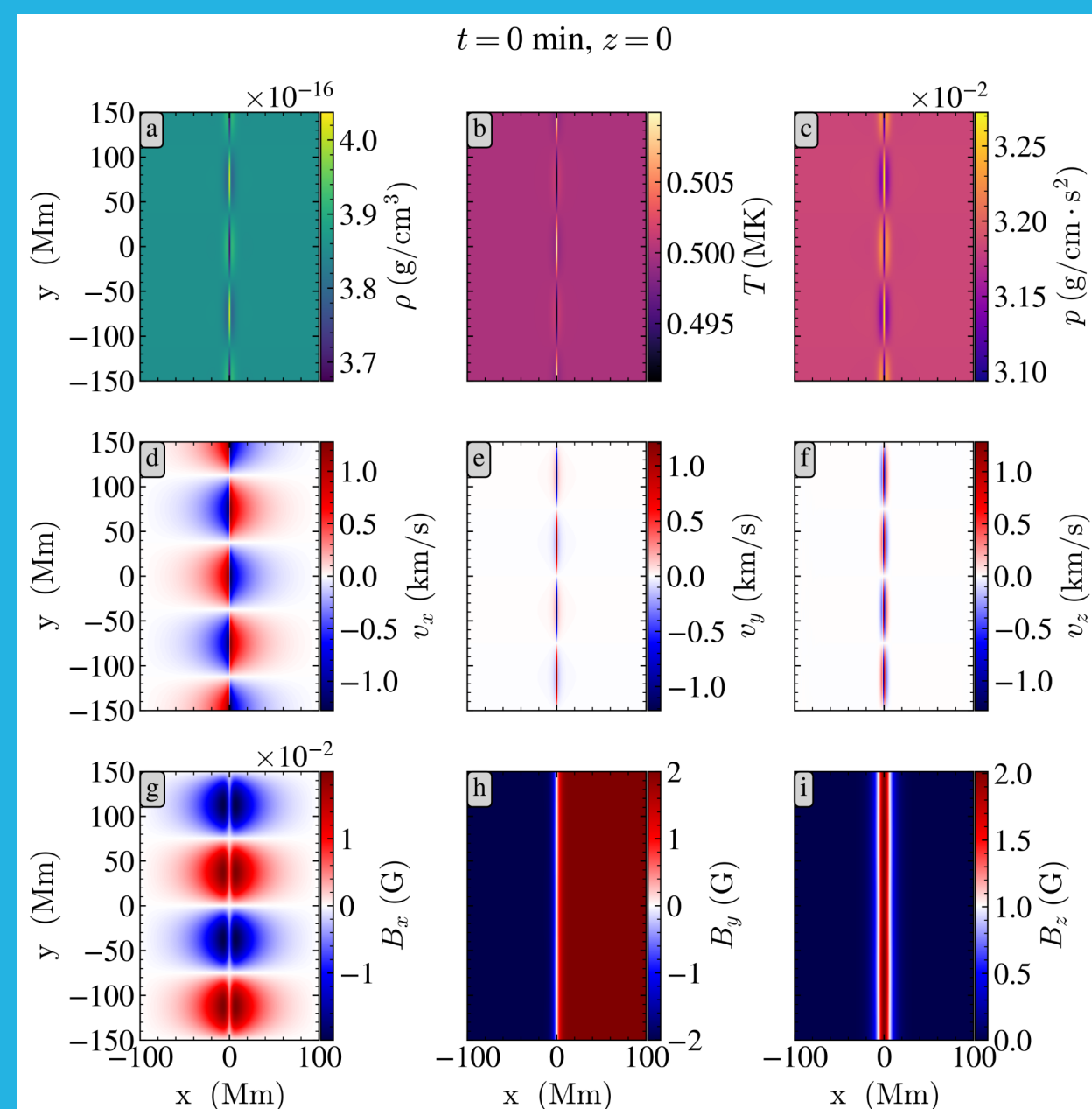
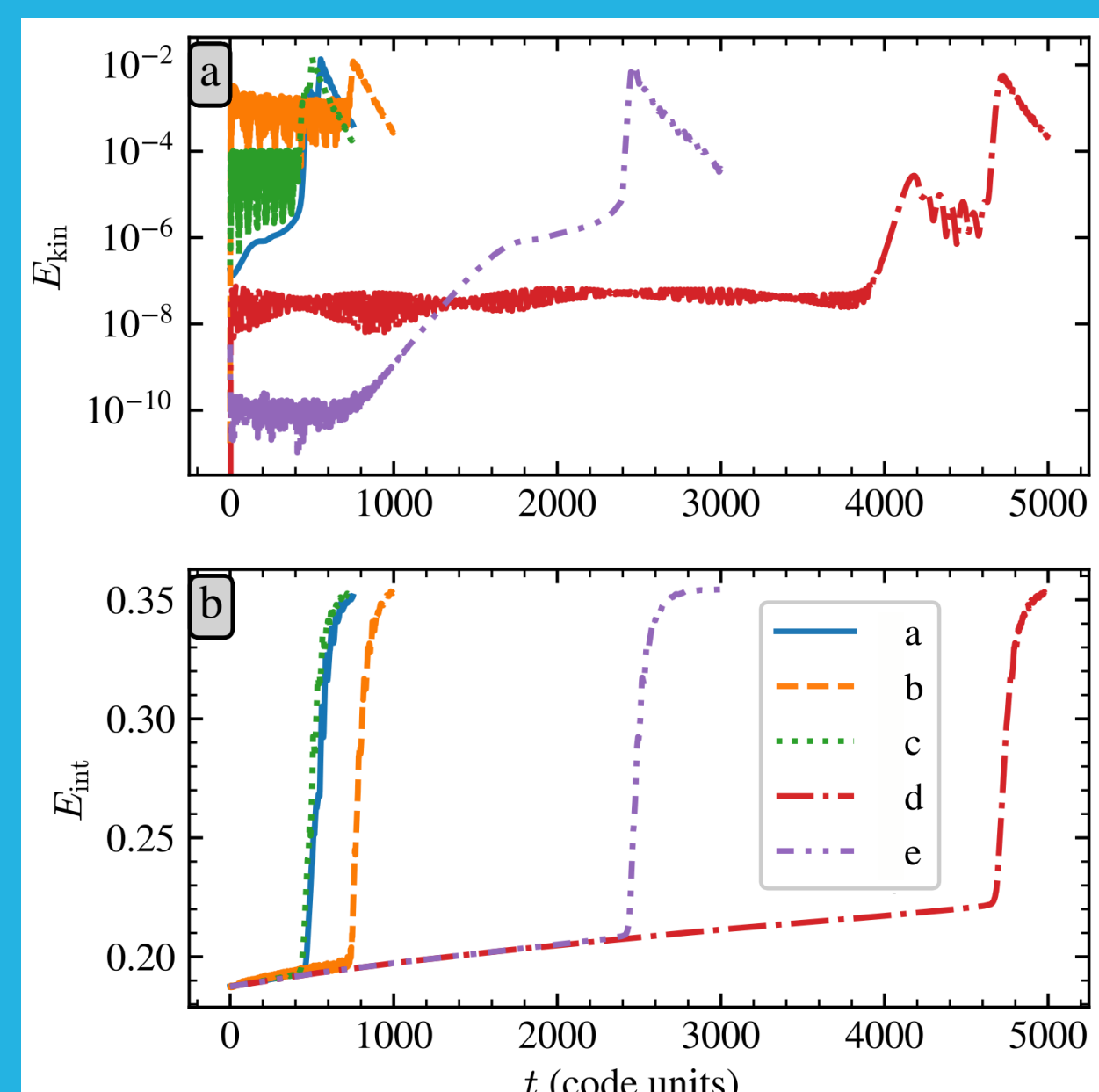


Figure: Harris current sheet in the corona perturbed by the tearing instability. [6]

magnetic field perturbation. There is thus a tangible advantage to perturbing with eigenmodes.

To facilitate introducing linear eigenmodes as initial perturbations in *MPI-AMRVAC* simulations, we developed the *GIMLI* framework [8]. This subpackage of *pylbo* allows the user to extract eigenmodes from their *Legolas* data and construct a superposition to be read by *MPI-AMRVAC*. To minimize room for user error in setting up equivalent states in *Legolas* and *MPI-AMRVAC*, *GIMLI* also allows the user to define their configuration as a set of *SymPy* expressions from which it generates user modules for both codes. Parameter files may be generated for both codes from code-specific dictionaries too.

Figure: Energy in Harris current sheet simulations perturbed with different initial perturbations. [5]



Simulation slice analysis

Though the *Legolas* code was initially designed to work with analytically prescribed initial states, the formatting of numerical arrays with *GIMLI* to be read by *Legolas*, and the addition of numerical derivative schemes, now makes it possible to analyze one-dimensional simulation slices. In [9], the authors performed 2D *MPI-AMRVAC* simulations of helmet streamers, i.e. large ray-like structures extending from the solar corona and forming long current sheets. The simulations were revisited in [10], where we investigated whether helmet streamer waves can be interpreted as fast body kink waves of the streamer slab.

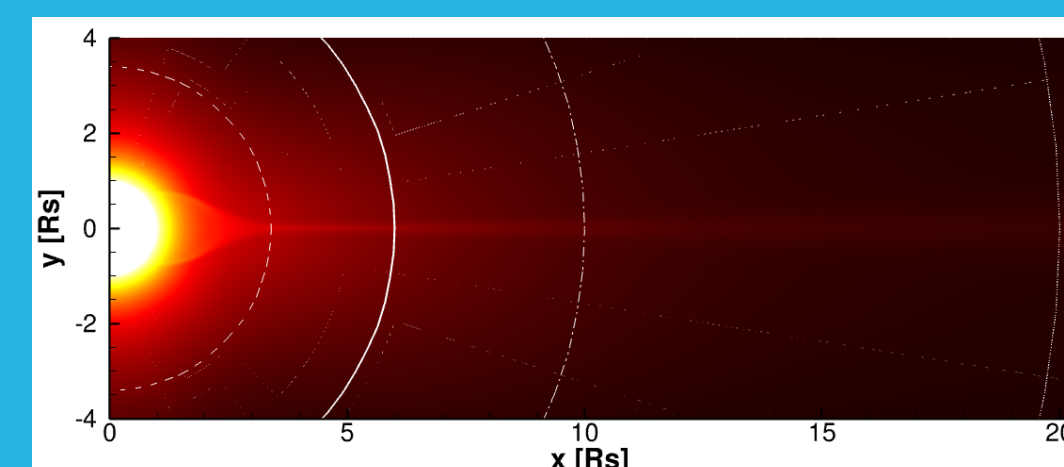


Figure: Radial slits taken from a helmet streamer simulation. [10]

For this analysis with *Legolas*, data was extracted from the 2D simulations along slits at a fixed radial distance, as shown in the figure above. The *Legolas* results for such a slit at 6 solar radii is shown on the right. Panel a shows the spectrum of the helmet streamer interpreted as a Cartesian slab model for a wavelength of 6 solar radii, with the first fast body kink mode marked with an orange diamond. The corresponding density perturbation amplitude is presented in panel b, with a 2D reconstruction of the density in panel c, confirming its identification as a body mode. The mode's phase speed was then calculated from the frequency and wave number, and compared to limited analytical models and simulation measurements of the streamer wave's phase speed.

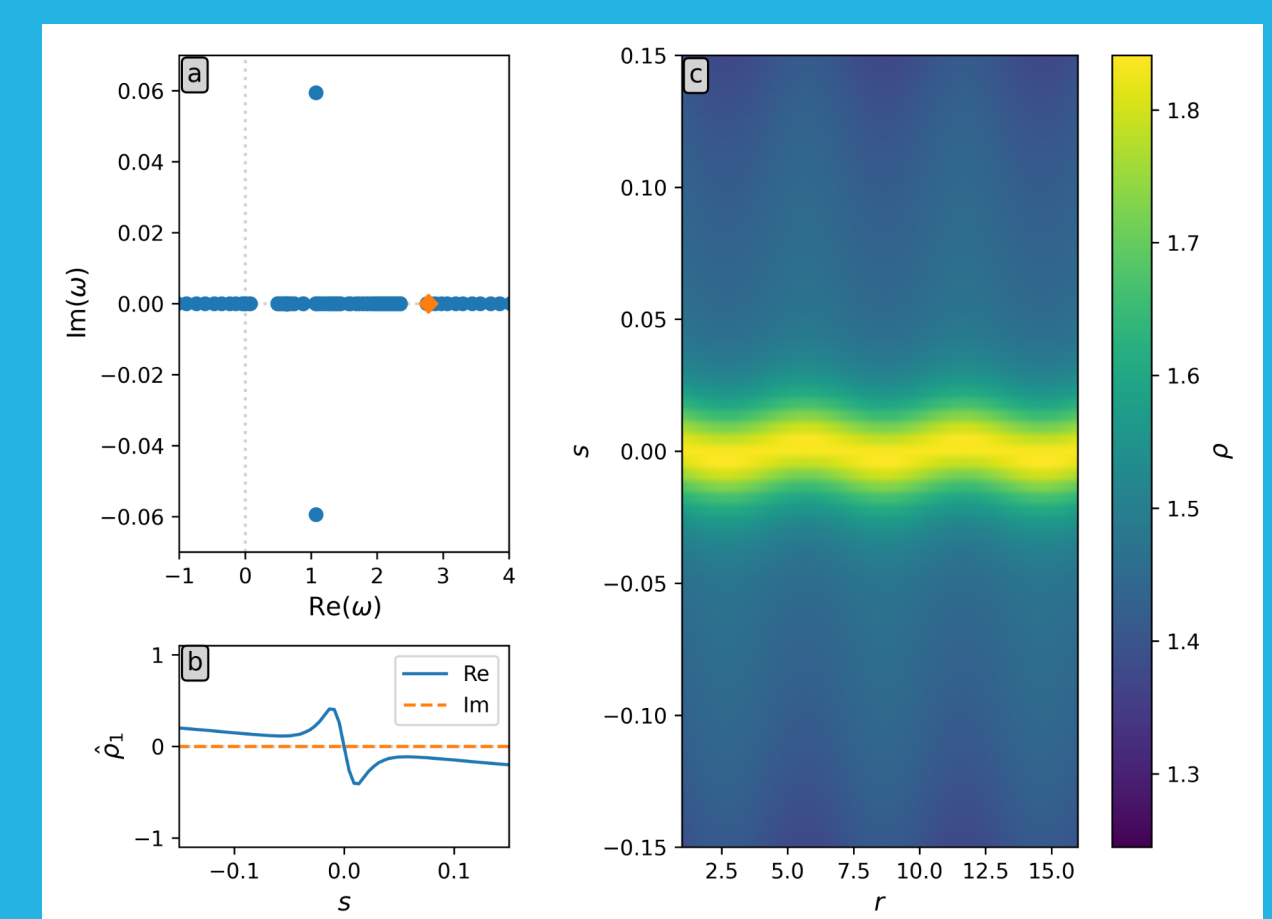
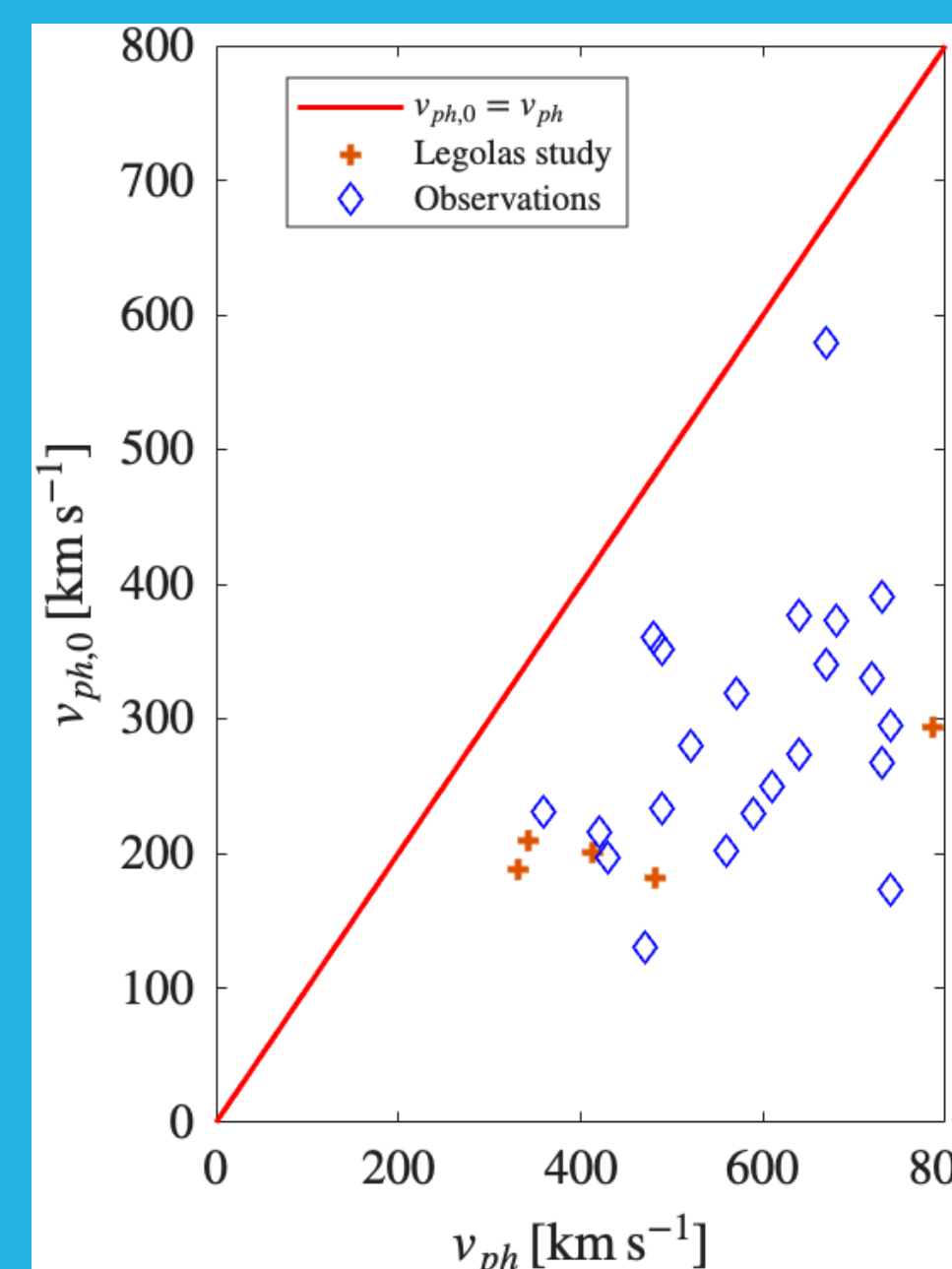


Figure: Legolas analysis of a simulation slit identifying the fast body kink mode. [10]



The *Legolas* analysis of the simulations slits was then repeated without the background flow to obtain the rest frame phase speed. The figure on the left shows the observer vs rest frame phase speed diagram combining the five slits analyzed with *Legolas* (crosses) and several observations of helmet streamer waves (diamonds). Overall, the considered cases are compatible with the observations, providing strong evidence that helmet streamer waves can be interpreted as fast body kink modes of these structures.

The interpretation of helmet streamer waves as fast body kink modes is promising for seismology. If rough shapes for the streamer profiles can be inferred from observations, alongside wavelength and bulk velocity estimates, iterative application of the *Legolas* code to find compatible phase speeds may yield estimates for the remaining coronal parameters.

Figure: Phase speed in observer vs rest frame: comparison of Legolas simulation analysis results to observations. [10]

References

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