Waves in a warm, ideal two-fluid plasma and model applications

J. De Jonghe* and R. Keppens

Centre for mathematical Plasma-Astrophysics, KU Leuven

*E-mail: jordi.dejonghe@kuleuven.be

1. Introduction

- *Observation.* Plasma physics textbooks focus on propagation parallel or perpendicular to the background magnetic field.
- *Consequence.* Wave labelling schemes identify waves at *all* propagation angles by their properties at parallel and perpendicular propagation.
- *Issue.* Parallel and perpendicular propagation are not representative of oblique angles because at these angles wave modes are allowed to cross, whilst all crossings are avoided at oblique angles. [1]
- *Solution.* A new unambiguous wave labelling scheme (**S**, **A**, **F**, **M**, **O**, and **X**) based on the frequency ordering at oblique angles.
- *Further observations.* Parallel/perpendicular crossings depend on the parameter regime [1], whistling behaviour occurs across all modes and all angles [2], and Cherenkov radiation is limited to a forward cone determined by the temperature.

2. Model and conventions

- *Setup.* Ideal, homogeneous ion-electron plasma at rest in a magnetic field.
- *Mathematical description.* Two-fluid description: ideal continuity, momentum, and energy equations for each species (ion/electron) and Maxwell's equations.
- *Further assumptions.* Charge neutrality and plane wave solutions.
- *Result.* Polynomial dispersion relation of sixth order in the squared frequency ω^2 and fourth order in the squared wavenumber k^2 (note: the frequency is normalised to the plasma frequency and the wavenumber with the combined skin depth).
- *Wave labelling.* The six forward-backward propagating wave pairs are labelled S, A, F, M, O, and X and are related to the slow (**S**), Alfvén (**A**) and fast (**F**) magnetohydrodynamic (MHD) waves, the (modified) electrostatic mode (**M**) and the ordinary (**O**) and extraordinary (**X**) electromagnetic modes.

3. Dispersion diagrams [1]

- *Perpendicular propagation.* **M** and **O** modes cross if the magnetisation exceeds a critical value, which depends only on the plasma's composition. The crossing can be expressed analytically.
- *Parallel propagation.* The amount of crossings, as well as which modes cross, depends on the magnetisation and the ratio of sound to Alfvén speed. There are six different physical regimes. The crossings can be approximated analytically or determined numerically.
- *Oblique propagation.* Any parallel/perpendicular crossings are replaced by avoided crossings (see Figure 1). [1]

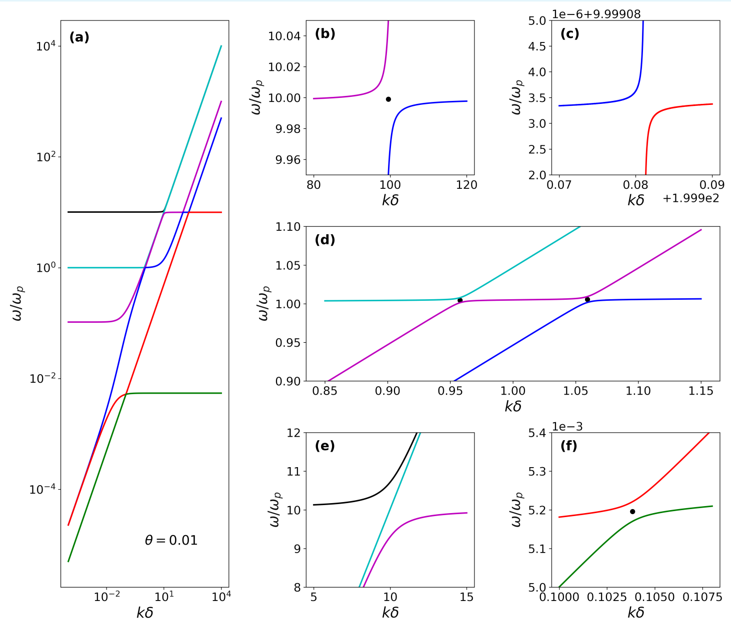

FIGURE 1: All crossings at parallel propagation are replaced with avoided crossings at a small angle. Black dots indicate the (numerical) crossings at parallel propagation. For panel (c), the dot is outside of the frame.

4. Group speed diagrams [1]

- Group speed can be computed at all angles, resulting in group speed diagrams.
- Avoided crossings are characterised by the reconnection of different branches. This is illustrated for the **M** and **O** mode in Figure 2.

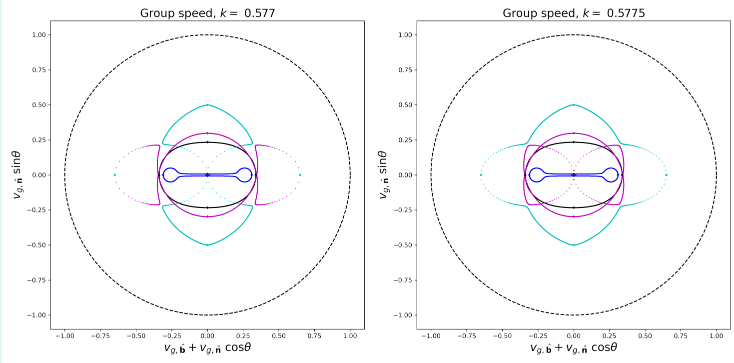

FIGURE 2: Group speed diagram before and after the **MO** avoided crossing.

5. Whistling behaviour [2]

- *Whistling behaviour.* Strong variations in group speed as a function of the frequency, such that different frequencies reach an observer at different times.
- Whistling behaviour occurs in *all* modes at *all* angles (e.g. Figure 3) in various frequency ranges (not all in the audible frequency range) and on different timescales.

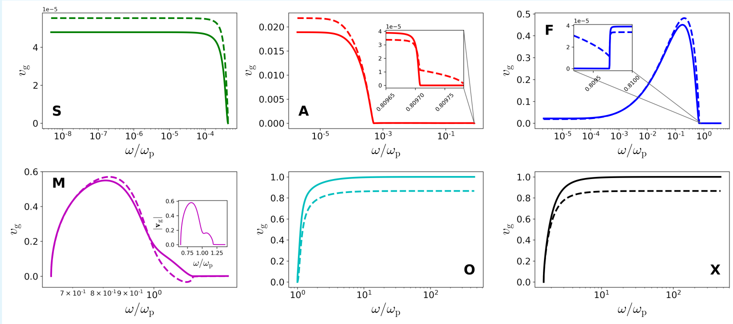

FIGURE 3: Group speed along the wavevector (solid $v_g \cdot \hat{e}_k$) and magnetic field (dash $v_g \cdot \hat{e}_B$) at a propagation angle $\theta = \pi/6$ for magnetosphere parameters.

6. Cherenkov radiation

- *Setup.* A laser beam interacting with a magnetised ion-electron plasma, where the laser is perpendicular to the magnetic field
- *Effect.* Coupling between laser and modified electrostatic **M** mode through the emission of Cherenkov radiation at an angle to the laser beam (see Figure 4)
- *Observation.* Emission is restricted to a cone since the **M** mode approaches a constant (temperature-dependent) index of refraction (larger than 1) for $k \to \infty$

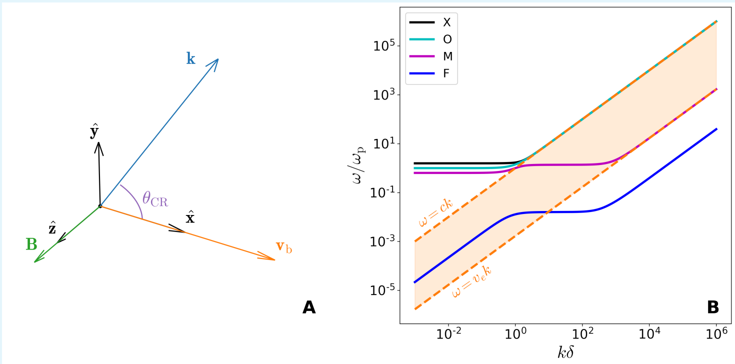

FIGURE 4: (A) Setup with a laser beam perpendicular to the magnetic field. (B) The shaded region indicates the region of interaction with the **M** mode.

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References

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[2] De Jonghe, J., & Keppens, R. (2021). Two-fluid treatment of whistling behavior and the warm Appleton-Hartree extension. *J. Geophys. Res. Space Phys.*, **126**, e2020JA028953.